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The construction of exact solutions in the floating-plate problem☆

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Abstract

It is shown that it is possible to construct, by an inverse method, exact solutions of the problem of the flexural-gravitational oscillations of a floating elastic plate. The results obtained are used to check the accuracy of numerical solutions of the problem. It is shown that the numerical algorithm given in Ref. [Khabakhpasheva TI. The plane problem of an elastic floating plate. In Continuum Dynamics. Inst. Gidrodinamiki SO Ross Akad Nauk 2000;**16**:166–9.], predicts, with high accuracy, the values of the amplitudes of the oscillations of the plate and the distributions of the bending moments and hydrodynamic pressure for a wide frequency range.

The problem of the oscillations of an elastic plate on the liquid surface due to the action of an incident wave or an external load has been actively investigated during the last decade.[2,3] Previously, the problem of the flexural-gravitational oscillations of plates was considered in relation to the investigation of the behaviour of ice floes.[4,5]

Large floating structures (for example, an airfield) have the form of a plate, elongated in one direction, which enables a two-dimensional formulation to be used to describe them. For a plate of finite length, several numerical algorithms have been proposed in this formulation.[1,6–9] The method of normal modes was used in Refs. 6,7 in which the deflection of the plate is represented in the form of an expansion in forms of its free oscillations in a vacuum; in this case, the interaction of the plate with the liquid is described using added masses, defined individually for each oscillation mode. The elements of the added mass matrix was determined[6] from the hydrodynamic part of the problem by the method of decomposing the flow region, while in Ref. 7 they were calculated explicitly. The method of decomposing the flow region was also used to solve the more general problem of the oblique incidence of a surface wave on a plate.[8] The solution of the problem of a floating plate, ignoring its inertial properties, was reduced, by the Wiener-Hopf method, to an infinite reducible system of differential equations.[9] A method of solving the problem was proposed in Ref. 1, in which the elastic and hydrodynamic characteristics are expanded in different basis functions: the hydrodynamic pressure is represented in the form of a Fourier series, while the deflection of the plate is sought in the form of an expansion in functions corresponding to the "response" of the plate to the pressure, specified by trigonometric functions.

Approximate solutions, constructed using the above algorithms, were compared both with one another and with existing experimental data in Ref. 6, in order to check their accuracy. The convergence of each numerical algorithm was also investigated. This attention to the accuracy of the numerical solutions is due to the fact that the planned dimensions of floating structures are very large, and it is difficult to satisfy criteria of similarity when making experimental investigations. In this situation the results of theoretical and numerical modelling become more reliable.[2] However,

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a comparison of the numerical results obtained by different methods does not enable one to determine which of the algorithms is better. If the results are different, it is not clear which of them is more accurate. To choose an adequate numerical algorithm it is necessary, first of all, to investigate the mathematical model and the features of its solution. Such an investigation has not yet been carried out for the problem of a floating plate, due to the complexity of the problem: the hydrodynamic problem of the motion of the liquid and the elastic problem of the oscillations of the plate have to be solved simultaneously.

Differences in the numerical results, obtained by existing methods,[1,6–9] emerge in the high-frequency action on a plate, which corresponds to short incident waves. The reason for these differences is so far unclear. It is to be hoped that a comparison of the numerical results with the accurate solutions, suitable for all frequencies of the action, will enable the area of applicability of each of the algorithms used to be determined.

In numerical investigations of the hydroelastic behaviour of a floating plate, the main attention has been devoted to time-periodic oscillations of the plate on waves. Obviously, without an accurate solution of this problem it is impossible to expect acceptable results for other important practical problems, such as, for example, the problem of the behaviour of a floating plate acted upon by a specified external load. Nevertheless, exact test solutions turned out to be constructed precisely for the problem of the hydroelastic oscillations of a plate acted upon by an external periodic load.

Exact test solutions have been constructed by an inverse method. In this method, the distribution of the hydrodynamic pressure along the plate is assumed to be specified, and the corresponding form of the boundary of the liquid is found from the solution of the hydrodynamic part of the initial problem. By identifying the deflection of the plate with the form of the liquid surface in the region where the hydrodynamic pressure differs from atmospheric and using the equation of motion of the plate, one can recover the distribution of the external load along the plate. The corresponding solution can be obtained with any prescribed degree of accuracy and can be used to test the numerical algorithms.

Note that, due to the need to satisfy the boundary conditions on the plate edge, the initial distribution of the hydrodynamic pressure along the plate cannot be chosen arbitrarily. These conditions are easily satisfied in the case of the two-dimensional problem, which is used below to illustrate the procedure for constructing exact solutions by the inverse method and to test the numerical method proposed previously in Ref. 1.

1. Formulation of the problem

We will consider the plane linear problem of the behaviour of an elastic plate, floating on the surface of a liquid of finite depth (Fig. 1). The time-periodic oscillations of the plate are due to an external load which acts on the plate with a frequency ω and has a small amplitude B. The plate is of constant density ρ_p and thickness h, and its dynamic stiffness is equal to EJ (E is the modulus of elasticity, J is the moment of inertia of the plate cross-section, and $J = h^3/12$ for a beam of uniform thickness h). The draught of the plate $d = \rho_p h/\rho$ (ρ is the liquid density) is assumed to be small compared with its overall length 2 L and the depth of the liquid H. The motion of the plate is described by the Euler equation for a beam. The plate edges are stress-free. It is required to determine the deflection of the plate, the stress distribution in it and the distribution of the hydrodynamic pressure on the lower side of the floating plate as a function of the plate parameters and the frequency of the external action. The external load $Bq(x', t'\omega)$, where $|x'| < L$ and $|q(x', t'\omega)| \le 1$, is symmetrical about the middle of the plate, which is chosen as the origin of a Cartesian system of coordinates $x'Oy'$ (the prime indicates dimensional variables). The flow region $-H < y' < 0$ is limited downwards by the horizontal bottom $y' = -H$. The upper boundary $y' = 0$ when $-L < x' < L$ is occupied by the floating plate, and when $|x| > L$ it is the free surface. The liquid is assumed to be ideal, ponderable and incompressible,

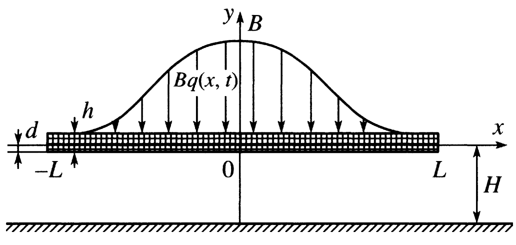

Fig. 1.

and its flow is symmetrical and vortex-free. In the linear theory, the liquid flow is described by the velocity potential $\phi'(x', y', t')$, while the oscillations of the plate are described by its normal deflection $w'(x', t')$, where t' is the time.

We will use the following dimensionless variables below

$$x = \frac{x'}{L},\quad y = \frac{y'}{L},\quad t = \omega t',\quad p = \frac{p'}{B},\quad w = \frac{\rho g}{B}\omega',\quad \phi = \frac{\rho g}{\omega BL}\phi',\quad M = \frac{M'}{2LdB}$$

Here g is the acceleration due to gravity and M' is the bending moment.

The dimensionless flow velocity potential $\phi(x, y, t)$ is the solution of the following boundary-value problem for the Laplace equation

$$\phi_{xx} + \phi_{yy} = 0,\quad -\infty < x < +\infty,\quad -H_0 < y < 0$$

$$y = -H_0{:}\ \phi_y = 0,\quad H_0 = H/L$$

$$y = 0{:}\ \phi_y = \eta_t,\quad \gamma\phi_t + \eta = 0,\quad |x| > 1;\quad \phi_y = w_t(x, t),\quad |x| < 1;\quad \gamma = L\omega^2/g$$

The equations $y = \eta(x, t)$ describes the form of the free boundary. The deflection of the plate $w(x, t)$ is the solution of the initial-boundary-value problem for the Euler equation for a beam

$$\alpha w_{tt} + \beta w_{xxxx} = p(x, t) - q(x, t),\quad |x| < 1;\quad w_{xx}(\pm 1, t) = 0,\quad w_{xxx}(\pm 1, t) = 0$$

where $p(x, t)$ is the distribution of the hydrodynamic pressure along the plate, which is determined by the linearized Cauchy-Lagrange integral

$$p(x, t) = -\gamma\phi_t(x, 0, t) - w(x, t)$$

The function $q(x, t)$ describes the distribution of the external load along the plate; $\alpha = \gamma d/L$ and $\beta = EJ/(\rho g L^4)$ are dimensionless parameters of the problem.

Note that the initial conditions are not specified, since we are interested in time-periodic solutions of the problem.

In the case of a periodic external load, the solution will be sought in the form

$$\phi(x, y, t) = \mathrm{Re}[ie^{it}\Phi(x, y)],\quad (w(x, t), p(x, t), q(x, t)) = \mathrm{Re}[e^{it}W(x), P(x), Q(x)]$$

The new complex-valued required functions $\Phi(x, y)$, $W(x)$ and $P(x)$ satisfy the following equations and boundary conditions

$$\Phi_{xx} + \Phi_{yy} = 0,\quad -\infty < x < +\infty,\quad -H_0 < y < 0 \tag{1.1}$$

$$y = -H_0{:}\ \Phi_y = 0 \tag{1.2}$$

$$y = 0{:}\ \Phi_y = \gamma\Phi,\quad |x| > 1;\quad \Phi_y = W(x),\quad |x| < 1 \tag{1.3}$$

$$P(x) = \gamma\Phi(x, 0) - W(x),\quad |x| < 1 \tag{1.4}$$

$$\beta W^{IV} - \alpha W = P(x) - Q(x),\quad |x| < 1 \tag{1.5}$$

$$W''(\pm 1) = 0,\quad W'''(\pm 1) = 0 \tag{1.6}$$

Moreover, the flow potential $\Phi(x, y)$ must be continuous up to the boundary and describe departing waves at infinity.

We will convert relations (1.1)–(1.4), which are the hydrodynamic part of the problem. Application of a Fourier transformation to Eq. (1.1), taking condition (1.2) into account, leads to the formula

$$\Phi^F(\xi, y) = \Phi^F(\xi, 0)\frac{\mathrm{ch}[\xi(y + H_0)]}{\mathrm{ch}(\xi H_0)}$$

The following equation is then satisfied on the upper boundary

$$\Phi_y^F(\xi, 0) = \xi \operatorname{th}(\xi H_0)\Phi^F(\xi, 0)$$

and conditions (1.3) and (1.4) can be rewritten in the form

$$\Phi^F(\xi, 0) = -\frac{1}{\xi \operatorname{th}(\xi H_0) - \gamma}\int_{-1}^{1} P(x_0)\exp(-ix_0\xi)dx_0$$

Applying an inverse Fourier transformation, we arrive at the relation

$$\Phi(x, 0) = -\frac{1}{2\pi}\int_{-1}^{1} P(x_0)K(x - x_0)dx_0; \quad K(z) = \int_l \frac{\exp(i\xi z)}{\xi \operatorname{th}(\xi H_0) - \gamma}d\xi$$

The contour of integration l passes along the x axis, passing round the pole $\xi = k$ from above, and the pole $\xi = -k$ from below, which is required in order to satisfy the conditions at infinity. Here k is the dimensionless wave number – a real positive solution of the dispersion equation $k \operatorname{th}(kH_0) = \gamma$. The use of the theorem of residues to calculate the inner integral enables us to determine the kernel $K(z)$ in the form of a series

$$K(z) = -2\pi i\frac{k\exp(-ik|z|)}{H_0(k^2 - \gamma^2) + \gamma} + 2\pi\sum_{j=1}^{\infty}\frac{s_j\exp(-s_j|z|)}{H_0(s_j^2 + \gamma^2) - \gamma}; \quad s_j = \frac{\pi j - \delta_j}{H_0}$$

where δ_j is the solution of the equation $\delta_j = \operatorname{arctg}(\gamma H_0/(\pi j - \delta_j))$ $(j \geq 1)$.[10,11]

By substituting the representation for $\Phi(x, 0)$ into condition (1.4) we can reduce the hydrodynamic part of the problem to the solution of an integral equation in the distribution of the hydrodynamic pressure $P(x)$ along the plate[1]

$$P(x) + \frac{\gamma}{2\pi}\int_{-1}^{1} P(x_0)K(x - x_0)dx_0 = -W(x) \tag{1.7}$$

Hence, it is required to determine the functions $W(x)$ and $P(x)$, which satisfy integral Eq. (1.7), differential Eq. (1.5) and boundary conditions (1.6) for a given function $Q(x)$. Moreover, it is of practical interest to represent the distribution of the bending moments, which are calculated from the formula

$$M(x) = -\frac{\beta L}{2d}W''(x)$$

In the general case, problem (1.5)–(1.7) is fairly complex, and its solution can only be obtained numerically. Below we will construct an exact particular solution of this problem using the inverse method.

2. The inverse method

The inverse method is based on the following idea. We will assume that the distribution of the hydrodynamic pressure $P(x)$ is specified. Then, the deflection of the plate $W(x)$ can be calculated using integral relation (1.7), while the external load $Q(x)$ is found from Eq. (1.5). It is should be noted that the function $Q(x)$ can be calculated with a prescribed degree of accuracy, since the deflection $W(x)$ is defined explicitly in terms of $P(x)$, and the external load $Q(x)$ is defined explicitly in terms of $W(x)$ and $P(x)$. Hence, in the inverse method, the function $P(x)$ is specified, while the functions $W(x)$ and $Q(x)$ are calculated from formula (1.7) and (1.5) respectively.

Note that the pressure $P(x)$ cannot be chosen arbitrarily. In fact, the free surface elevation when $|x| > 1$ and the deflection of the plate when $|x| < 1$ are equal to the normal derivative $\Phi_y(x, 0)$, and, in general, their values at the plate edges are not identical.[12] This means that the function $\Phi_y(x, 0)$ undergoes a discontinuity when $x = \pm 1$ and can be represented in the form

$$\Phi_y(x,0) = F(x) + CH(1-x^2); \quad C = \Phi_y(1-0,0) - \Phi_y(1+0,0) \tag{2.1}$$

where $H(x)$ is Heaviside's function, and the function $F(x)$ is continuous.

Substituting expression (2.1) into condition (1.4) and taking condition (1.3) into account, we obtain that, in the general case of symmetrical oscillations of the plate,

$$P(1) = P(-1) \neq 0$$

The solution of Laplace Eq. (1.1) with boundary conditions (2.1) and (1.2) has a singularity when $x \to \pm 1$ of the form $\Phi_x = O(\ln(1-x^2))$, whence it follows that

$$\Phi(x,0) - \Phi(1,0) = O((1-x)\ln(1-x))$$

Substituting the last asymptotic formula into condition (1.4) and taking into account the fact that $W(x) \in C^4(-1, 1)$, we obtain

$$P(x) - P(1) = O((1-x)\ln(1-x)), \quad 1-x \to +0$$

Hence, in order to calculate the derivatives $W''(\pm 1)$, $W'''(\pm 1)$ using relation (1.7) and to verify that conditions (1.6) are satisfied, we cannot to differentiate this relation and take the limit as $x \to \pm 1$ term by term. However, this is possible in the special case when

$$P(1) = P'(1) = P''(1) = P'''(1) = 0 \tag{2.2}$$

The last limitations enable Eq. (1.7) to be differentiated four times with respect to x and enable us to take the limit as $x \to \pm 1$ term by term. It can be shown that relations (2.2) ensure continuity of the liquid boundary at the plate edges, but $W(\pm 1) \neq 0$.

In order to satisfy boundary conditions (1.6), we will consider three continuous, even and real-valued functions $f_j(x)$ ($j = 1, 2, 3$), which satisfy conditions (2.2). We will represent the pressure $P(x)$ in the form of a linear combination of these functions

$$P(x) = \sum K_j f_j(x) \tag{2.3}$$

with as yet undetermined complex coefficients K_1, K_2 and K_3. Substituting expression (2.3) into Eq. (1.7), we obtain

$$W(x) = \sum K_j W_j(x) \tag{2.4}$$

where $W_j(x)$ are complex-valued, continuous, even functions, obtained from Eq. (1.7) by substituting $f_j(x)$ instead of $P(x)$. If we substitute expansions (2.3) and (2.4) into the beam Eq. (1.5) we obtain a formula for the external load

$$Q(x) = \sum K_j Q_j(x) \tag{2.5}$$

The functions $f_j(x)$ can be chosen in such a way that the integral in Eq. (1.7) for each of them can be evaluated analytically, in which case the accuracy of the calculations when using the operator method is guaranteed.

Representation (2.3) and conditions (1.6) lead to two equations for the complex coefficients K_j. The third equation follows from formula (2.5) and the normalization condition $\max|Q(x)| = 1$.

After calculating the coefficients K_j, expansions (2.3) and (2.4) give the exact solution of the initial problem for the external pressure specified by formula (2.5).

To check the accuracy of any numerical algorithm, we will return to the direct problem of hydroelasticity (1.5)–(1.7), where the distribution of the external load $Q(x)$ is specified in accordance with the solution of inverse problem (2.5), and we will compare the numerical values of the amplitudes of the pressure, the deflections and the bending moments with the exact solutions, which are given by formulae (2.3) and (2.4). Solving the inverse problem, and then the direct problem for different values of the parameter γ and different test functions $f_j(x)$, ($j = 1, 2, 3$) and comparing the results, we can judge the accuracy and particular features of the numerical method employed.

3. Numerical results

Test calculations were carried out for the conditions of the experiments described in Ref. 6 with a uniform narrow plate in a channel of depth $H = 1.1$ m. The stiffness of the plate $EJ = 471$ kg m^3/s^2, its length $2L = 10$ m and its thickness $h = 38$ mm. The draught of the plate $d = 8.36$ mm. For these dimensions and stiffness of the plate we obtain $\beta = 7.7 \times 10^{-5}$. The values of the remaining parameters of the problem are presented below as a function of the oscillation frequency ω

ω, s^{-1}	2.2	4.4	8.98	15.7
$\alpha \times 10^3$	4	16	69	210
γ	2.43	9.85	41.06	125.8
k	3.65	10.1	41.06	125.8

These values of the parameters (with the exception of the frequency of 15.7) were used previously in Refs. 1,10,11 to investigate the problem of the oscillations of an elastic floating plate acted upon by a regular surface wave of wavelength $4\pi L/k$.

The numerical method,[1,10,11] is based on expansion of the pressure along the plate and of the bending of the plate in terms of different basis functions, which enabled the solution of the hydrodynamic part of the problem to be simplified, and at the same time satisfy the conditions at the plate edges exactly. The numerical data were compared with the results obtained in other papers.[6–9] It was found that the distributions of the amplitudes of the deflections of the plate and of the stresses in it, obtained by different methods, are practically identical for medium and low frequencies of the incident wave ($\omega = 4.4$ s^{-1} and $\omega = 2.2$ s^{-1}). However, for high frequencies ($\omega = 8.98$ s^{-1}) the results differ considerably both for the deflections and for the stresses. The reason for these differences is not clear, and testing using the exact solutions may reveal the accuracy and areas of applicability of the methods employed by different researchers. This was done here for the method proposed previously in Ref. 1. The scheme proposed in Refs. 1,10, intended to describe the behaviour of an unclamped floating plate on waves, was modified to correspond to the change in the conditions under which the oscillations of the plate were excited.

In the inverse method, the following test functions were employed

$$f_1(x) = \cos^4\frac{\pi x}{2}, \quad f_2(x) = \cos^6\frac{\pi x}{2}, \quad f_3(x) = \cos^8\frac{\pi x}{2}$$

which were substituted into expression (2.2) to determine the external load $Q(x)$ according to formula (2.4). After this the direct problem was solved by the method described previously in Ref. 1, for the external load, given by this function $Q(x)$.

The calculations showed that at low frequencies ($\omega = 2.2$ s^{-1} and $\omega = 4.4$ s^{-1}) the pressure $P(x)$ and the external load $Q(x)$ differ considerably. The quantity $|P(x) - Q(x)|$, on the right-hand side of Euler equation, is large, and the results of direct and inverse numerical calculations are practically indistinguishable. This quantity decreases as the frequency increases. Consequently, to construct the numerical solution with a fixed relative accuracy it is necessary to increase the absolute accuracy of the calculations as the oscillation frequency increases. At very high frequencies, the required absolute accuracy of the calculations may become practically unattainable due to rounding errors. On the other hand, at high frequencies the effect of the compressibility of the liquid, the inertia of the rotation of the elements of the beam and the shear stresses in it become considerable, and the model itself may turn out to be unsuitable.

Testing of the algorithm proposed in Ref. 1 by the inverse method enabled us to reveal the following feature of this algorithm: the convergence of the series in the kernel $K(z)$ of Eq. (1.7) worsens as the frequency increases, and in order to calculate it with a given accuracy it is necessary to sum a larger number of terms. At a frequency of the external load below 5 s^{-1} it was sufficient to sum 100 terms of the series $K(z)$, whereas for a frequency of 15.7 s^{-1} it was necessary to take into account 5000 terms. It should be noted that, without exact test relations, it is practically impossible to reveal this feature of the direct numerical algorithm, since standard tests of the convergence of the numerical solution (an increase in the number of modes) and a comparison with the results of other researchers have not indicated this feature. The algorithm in Ref. 1 was modified taking this feature into account. Below we will only be concerned with the modified numerical algorithm.

In Figs. 2 and 3 we show the results of calculations using the direct and inverse method for fairly high frequencies of the external load, i.e. $\omega = 8.98$ s^{-1} (Fig. 2) and $\omega = 15.7$ s^{-1} (Fig. 3). At low frequencies the results of the direct

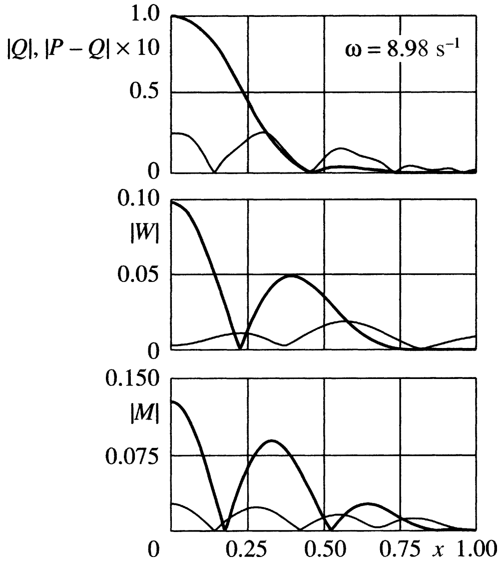

Fig. 2.

calculation are identical with the exact solution. The point $x = 0$ corresponds to the centre of the plate, while $x = 1$ corresponds to its edge (all the distributions are even). In the upper parts of Figs. 2 and 3 the heavy curve corresponds to the distribution of the modulus of the amplitude of the external load acting on the plate, $|Q(x)|$, while the thin curve corresponds to the tenfold modulus of the difference between the external load and hydrodynamic pressure, $|P(x) - Q(x)| \times 10$. Note that the thin curve corresponds both to the numerical and analytical solutions. Hence, the pressure is calculated with a high degree of accuracy by the direct method.

Correspondingly, for each case we present two curves for the distributions of the moduli of the amplitudes of the bending of the plate, $|W(x)|$, and the bending moment, $|M(x)|$. In Fig. 3 the results of the calculations using the direct method are shown by the dashed curve, while the results corresponding to the inverse method are represented by the continuous curve. Since, for a frequency $\omega = 8.98\,\text{s}^{-1}$ the results of the calculations using the direct and inverse methods are practically identical, in Fig. 2 the heavy curve shows the exact values of the moduli of the amplitudes of

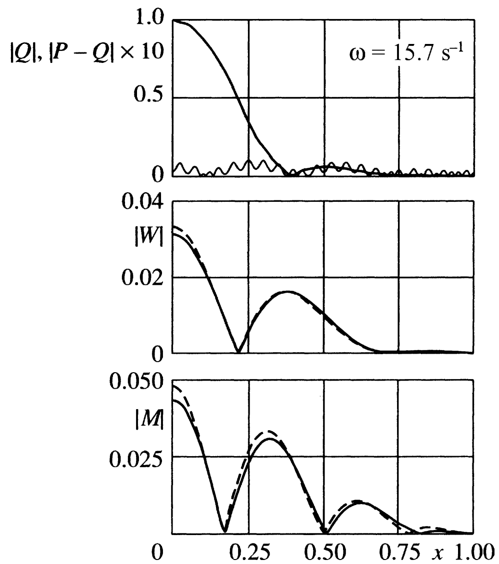

Fig. 3.

the deflections and moments, while the thin curve represents the tenfold moduli of the difference of these values and the results of the direct calculation.

Hence, testing of the exact relations shows that the method considered[1] enables one to carry out calculations with a high accuracy for all the frequencies of the external load considered. Particular attention is drawn to the good agreement between the results obtained for high frequencies, for which considerable disagreement with the results of the use of other methods is observed.[6,8]

The results obtained enable us to conclude that the method considered in Ref. 1 predicts both the hydrodynamic pressures on the floating elastic plate and the deformations of the plate quite well. It should, however, be noted that a comparison of the numerical solution with a particular exact solution does not enable any conclusion to be drawn regarding the accuracy of the numerical algorithm. Nevertheless, in the case when there is good agreement between these solutions, the suitability of the algorithm becomes more plausible.

The idea of constructing exact relations by the inverse method can be applied to other problems, when the construction of exact solutions of the direct problem is difficult or impossible.

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References

1. Khabakhpasheva TI. The plane problem of an elastic floating plate. In: *Continuum Dynamics*. Novosibirsk: Inst. Gidrodinamiki SO Ross. Akad. Nauk; 2000;Issue 16:166–9.
2. Proceedings of International Workshop on Very Large Floating Structures, Watanabe Y. editor, Japan: Hayama, 1996.
3. Proceedings of the 2nd International Conference on Hydroelasticity in Marine Technology, Kashiwagi M. et al. editors, Japan: Fukuoka; 1998.
4. Meylan M, Squire VA. Finite-floe wave reflection and transmission coefficients from a semi-infinite model. *J Geophys Res* 1993;**98**(C7):12.537–42.
5. Meylan M, Squire VA. The response of ice floes to ocean waves. *J Geophys Res* 1994;**99**(CI):891–900.
6. Chong Wu, Watanabe E, Utsunomiya T. An eigenfunction expansion-matching method for analyzing the wave-induced responses of an elastic floating plate. *Appl Ocean Res* 1995;**17**(5):301–10.
7. Korobkin AA. A numerical and asymptotic investigation of the plane problem of the hydroelastic behaviour of a floating plate on waves. *Zh Prikl Mekh Tekh Fiz* 2000;**41**(2):90–6.
8. Sturova IV. The oblique incidence of surface waves on an elastic strip. *Zh Prikl Mekh Tekh Fiz* 1999;**40**(4):62–8.
9. Tkacheva LA. The plane problem of the diffraction of surface waves by an elastic floating plate. *Izv Ross Akad Nauk MZhG* 2003;(3):131–49.
10. Khabakhpasheva TI, Korobkin AA. Hydroelastic behaviour of compound floating plate in waves. *J Engng Math* 2002;**44**(1):21–40.
11. Khabakhpasheva TI. The relation between the hydrodynamic and elastic parameters in the diffraction of surface waves by a floating plate. *Izv Ross Akad Nauk MZhG* 2003;(4):101–10.
12. Bessho M, Komatsu M. On hydrodynamical forces acting on a flat plate oscillating on water surface (Second Report). *KSNAJ (Kansai Society of Naval Architects Japan)* 1976;(163):67–74.

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